INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, Second Semester, 2003-04
Statistics - IV, Final Examination

(10) 1. Suppose we have a random sample $X_1, \ldots, X_n$ from a continuous distribution with unknown c.d.f. F. Consider testing $H_0 : F = F_0$, where F_0 is a completely specified c.d.f.
(a) What are the directional and non-directional Kolmogorov-Smirnov test statistics that are useful for this test and when are they used?
(b) Show that the Kolmogorov-Smirnov statistics mentioned in (a) are distribution free under the null hypothesis.

(10) 2. Suppose $X \sim$ Binomial(n, θ), where n is fixed but $0 < \theta < 1$ is unknown. Consider estimating θ under the loss $L(\theta, a) = \theta^{-1}(1-\theta)^{-1}(\theta - a)^2$.
(a) Show that $\delta_c(X) = cX$ is inadmissible if $c > 1/n$.
(b) Find the Bayes estimator of θ with respect to the prior $\pi(\theta) \propto \theta^{3/2}(1-\theta)^{3/2}$, $0 < \theta < 1$.

(10) 3. Suppose $X \sim$ Poisson(θ), where $\theta > 0$. Consider $L(\theta, a) = (\theta - a)^2/\theta$, where $a \geq 0$. Show that δ_0 defined by $\delta_0(x) = x$ is minimax.

(10) 4. Consider a two-person, zero-sum game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	4	6	0	0
θ_2	0	5	4	2

(a) Form the risk set.
(b) Find the set of all minimax strategies.

(10) 5. Consider a two-person, zero-sum game which is strictly determined.
(a) Explain why player I should use the maximin strategy and player II, the minimax strategy, if they are intelligent.
(b) Solve the game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	4	6	0	0
θ_2	0	5	4	2
θ_3	0	4	1	3